Simulation Modeling Using Markovian Decision Theory on Cash Flow Analysis of Central Bank of Nigeria

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Abstract

This research aims to investigate the cash flow analysis of the Central Bank of Nigeria using Markovian decision theory. The specific objectives include determining the (i) cash flow solvency ratio, (ii) cash flow adequacy ratio, (iii) sufficiency ratio, (iv) cash flow profitability ratio, and (v) estimating the optimal policy of cash flow ratios performance in CBN. The identified problems are the effects of (i) insolvency, (ii) inadequacy, (iii) insufficiency, and (v) Unprofitability on Central Bank of Nigeria (CBN) cash flows performance.The methodology involves a research design tailored toward collecting, arranging, and determining cash flow data for model prediction and optimization. The Markov chain is introduced as an operator to evaluate the distribution of cash flow ratios in the long term, using initial state vectors and state transition probabilities for forecasting behavior. Data validation is performed using graphical and Pearson moment correlation coefficient methods. The pre-model analysis of CBN cash flows problem during the period of January 2012 to December 2017 identifies six finite current states. State-2012 cash flows performance was exceptional (above the zero line), reflecting 100%, while State-2013 and 2015 reflect 75%, and State-2014, 2016 & 2017 reflect 50% healthy cash flows status. The model results introduce the Markovian Cash Flow Ratios Monitoring Curve (MCFRMC), specifying the minimum values for healthy status. The research explores the present status of cash flow ratios, presenting forecasted ratios in the form of an optimum policy or solution. Pearson moment correlation coefficient validation of the prototype and model results in a coefficient of 1.0, indicating a 100% higher performance of the model. Further research reveals strategic cash inflows policy allocation to the cash flows indicators, with (i) operational activity receiving 24%, (ii) investment activity receiving 38%, and (iii) financial activity receiving 38%. The optimal cash outflow strategy reveals that operational and financial activities tend towards 0%, while investment cash outflow tends towards 100%. In conclusion, the research suggests that the model developed can serve as a forecasting, monitoring, and allocation tool, aiding CBN operators in projecting preventive action plans against inflation and financial instability. The recommendation is made for CBN to employ inventory models like the Markovian decision model in monitoring and allocating cash flows.

The efficiency of Markov in predicting long-run behavior is acknowledged, extending its applicability to areas such as stock market analysis and manpower planning.

Keywords: *Cash Flow Analysis, Markovian Decision Theory, Cash Flow Ratios, Central Bank of Nigeria (CBN), Optimal Policy*

1.1 Background to the Study

This journal delves into the profound significance of decision-making across various levels, spanning individual, organizational, societal, governmental and Engineering domains. At the core of this exploration is an emphasis on the role of decision-making in shaping cash flow, a critical component of national economies. The influence of central banks in economic decision-making is highlighted, recognizing their substantial impact on economic agents and overall macroeconomic performance [7]. The journal further investigates the application of Markov analysis, a probabilistic technique, to model systems characterized by probabilistic transitions between states. An in-depth discussion on Markov chains, both discrete and continuous, elucidates their versatile applications in diverse fields, including economics, finance, and decision-making processes [22][17]. The latter part of the analysis explores Markovian decision processes, tracing their historical development and discussing their applications in optimizing system operations. The journal concludes by addressing the collective impact of structural inadequacies, exchange rate fluctuations, and various economic indicators on cash flow within the Nigerian economy [9].

1.2 Objectives of the Study

The study aims to explore simulation modeling through Markovian decision theory for analyzing the cash flow of the Central Bank of Nigeria. The specific objectives are to:

(i) Assess the impact of solvency ratio on CBN.
(ii) Examine the influence of Net Cash Flow Adequacy Ratio on CBN.
(iii) Analyze the Net Cash Flow Sufficiency Ratio on CBN.
(iv) Evaluate the Profitability of Cash Flow Ratio on CBN.
(v) Investigate the optimal policy for Cash Flow Ratios using Markovian Decision Theory.

1.3 Significance of the Study

The statement of cash flows has been a required part of annual financial statement for more than decades, several studies have suggested a comprehensive set of cash flow ratios with the potential to evaluate financial performance of CBN and application of simulation modeling using Markovian decision theory as superior optimization solution for the cash flow management. This study will provide an insight into management policies, performance and apparent priorities with Markov chain. The knowledge of the underlying characteristics of the cashflow across different sources, policies and reforms will benefit investors and policy makers [1]. It will enable the investors to improve their investment and risk management strategies. For instance, investors will understand to what extent the cashflow is or is not efficient, whether there are high levels of bubbles in the cashflow which will distort Nigerian economy [1]. This study will also enable the Central Bank of Nigeria and/or financial policy makers to improve the overall performance and operations of cashflow, by implementing policies that will ultimately make the cashflow more efficient, less prone to bubbles, and less volatile, for instance [1]. By using appropriate statistical and empirical cashflow models to study the issues and characteristics of the cashflow, this research contributes to the literature debt/base on quantitative modelling of Nigerian cashflow analysis [7].

1.4 Scope of the Study

This research focuses on the simulation modeling of Markovian decision theory in the context of cash flow analysis within the Nigerian economy from 2012 to 2017, covering a six-year horizon referred to as the current states. The selection of this time frame is justified by the availability of data and aims to ensure a stable and healthy financial position in the cash flow of the Central Bank of Nigeria. The objective is to conduct a comprehensive assessment of the quality of analysis in cash flow management [8][9].

2.1 Review of Related Literature

The literature review underscores a substantial gap in empirical literature related to the underutilization of simulation methods, particularly in the realm of management and economics operations. This observation aligns with findings by [22], who highlighted the slow adoption of simulation methods in management research [1]. The study under review specifically addresses this gap by employing Markovian chain simulation for modeling CBN cash flow ratios, a methodology grounded in the recognition of the complexities inherent in managerial and organizational behavior [22] [5]. This systematic approach is justified by drawing parallels with prior research that successfully applied simulation techniques in the domains of strategic management and organizational performance [21][20][16][4][9]. The decision to utilize the Markov Chain model is supported by its one-stage dependence of events, as explained by [19], and its demonstrated efficacy in understanding regime-switching behavior in financial time series, a concept pioneered by [13] and further explored by [12]. The significance of calculating the long-run distribution of regimes using the Markov chain, a principle mirrored in the research approach where present cash flow ratios are compared with forecasted optimum solutions [6]. The study also draws inspiration from diverse applications of Markov decision processes (MDPs), such as in the design of autonomous intelligent agents for forest fire fighting[15], managed Conjunctively Competitive Anambra and Imo River Basin and Dam Projects [10][11], GPU-based decision-making processes [3], and dynamic optimization of network operations[14]. Additionally, the literature review alludes to the stochastic nature of wireless sensor networks [18], providing a rationale for applying MDP in the analysis of cash flow ratios, considering them as stochastic systems influenced by randomness in the monitored environment (8). This comprehensive literature review not only identifies the existing gap but also positions the current research within a broader context of simulation methods, Markov Chain models, and Markov decision processes [1][2].

3.1 Methodology and Research Design

The research methodology and design involved collecting, organizing, and determining cash flow ratio data for model prediction and optimizing cash flow processes. The collected CBN data included cash inflow, operational activity, investment activity, financial activity, cash outflow, and net cash flow. Subsequently, the data were processed to derive variables such as Solvency ratio (SR), CBN Net Cashflow Adequacy Ratio (NTCFAR), CBN Cash Flow Sufficiency for Current Activities (CFSFCA), and CBN Profitability Cash Flow Ratio (PCFR). These ratios serve as indicators of the Central Bank of Nigeria's (CBN) cash flow health, representing the state of nature (y-variables) transitioning over a periodic interval (x-variables) for the years 2012 to 2015, organized in a 4 x 4 matrix for analysis as stated below:

From: Present	To:	Future		
Index	SR	NTCFAR	CFSFCA	POCFR
SR	X11	X12	X13	X14
NTCFAR	X21	X22	X23	X24
CFSFCA	X31	X32	X33	X34
POPCFR	X41	X42	X43	X44

Let the scalar quantity of the Arrangement above be:

Hence, let the vector quantity represented as: ∏1, ∏2, ∏3, ∏4.

Therefore, the objective function represented as

Recall: лP = л

Therefore, the objective function stated below:

$$\begin{pmatrix} \prod 1 \prod 2 \prod 3 \prod 4 \end{pmatrix} \times \begin{pmatrix} X11 & X12 & X13 & X14 \\ X21 & X22 & X23 & X24 \\ X31 & X32 & X33 & X34 \\ X41 & X42 & X43 & X44 \end{pmatrix} = \begin{pmatrix} \prod 1, \\ \prod 2, \\ \prod 3, \\ \prod 4. \end{pmatrix}$$ Equation 3.1

The product matrix above represents policy iteration values for decision-making. The Markovian decision in this work applies dynamic programming to solve a stochastic decision process with a finite number of stages, characterized by transition probabilities in a Markov chain. The reward structure is defined by a matrix indicating revenue or cost ratios between stages. Both transition and cost matrices depend on decision alternatives. The cash flow management problem aims to find the optimal policy (∏1, ∏2, ∏3, and ∏4) maximizing expected cash flow ratios over finite and infinite stages.

4.1 Data Analysis, Results and Optimization

This section encompasses the processes of data analysis and optimization for the model. It involves determining the Cashflow ratio status aligned with various research objectives in the context of CBN cash flow analysis. When planning CBN activities, declaring objectives is crucial to gauge efforts directed toward their achievement, serving as criteria for measuring the anticipated outcomes and future prognosis.

The primary objectives in CBN cashflow analysis include: (i) Solvency ratio (SR) of CBN Cashflow. (ii) CBN Net Cashflow Adequacy Ratio (NTCFAR). (iii) CBN Cash Flow Sufficiency for Current Activities (CFSFCA). (iv) CBN Profitability of Cash Flow Ratio (POCFR).

(i) **2012 Net Cashflow Analysis**

Table 4.1: 2012 Net Cashflow Analysis

DESCRIPTION	INFLOW	OUTFLOW	NET Cash flow
Net CBN Cash Inflow			46,783.20
Net CBN Operational Activities	45,071.43	34,441.62	10,629.81
Net CBN Investment Activities	1,461.77	0.57	1461.2
Net CBN Financial Activities	250	811.2	-561.2
Net CBN Cashflow	46,783.20	35,253.39	11,529.81
Net CBN Cash outflow			36,494.83

Table 4.2: Results and Discussions

S/n	Cashflow Ratio Description	CBN Ratio Nomenclature	Ratio Values	Remarks
1	The Solvency Ratio	SR	1.327055	Indicates money available to take care of obligation. Formula: [Inflow/outflow]
2	Net cash flow adequacy Ratio	NTCFAR	18648.79	Indicates money available for investment. Formula: [Net operation activity/Investment outflow]
3	Cash flow sufficiency Ratio	CFSFCA	13.10381	Indicates money available for payment of debt and liabilities. Formula: [Net Operation/Activity outflow]
4	Profitability of cash-flow Ratio	POCFR	0.031233	Indicates sufficient amount net Profit receive by CBN. Formula: [Net Investment/Cash inflow]

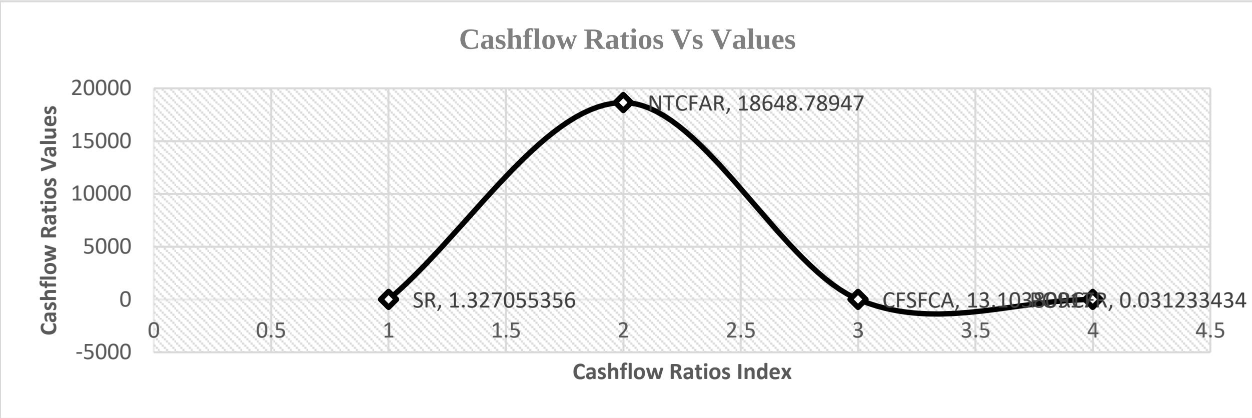

Figure 4.1: Graphical Representation of Cashflows ratios of State-2012

(ii) 2013 Net Cashflow Analysis

Table 4.3: 2013 Net Cashflow Analysis

DESCRIPTION	INFLOW	OUTFLOW	NET Cash flow
Net CBN Cash Inflow			41,070.29
Net CBN Operational Activities	40064.74	41,763.87	-1,699.13
Net CBN Investment Activities	1005.55	0	1005.55
Net CBN Financial Activities	0	549.23	-549.23
Net CBN Cashflow	41,070.29	42,313.10	-1,242.81
Net CBN Cash outflow			42,313.10

Table 4.4: Results and Discussion

S/n	Cashflow Ratio Description	Cashflow Ratio Index	Ratio Values	Indication
1	The solvency ratio within the year under consideration:	SR	0.970628	Indicates money available to take care of obligation. Formula: [Inflow/outflow]
2	Net cash flow adequacy ratio for current activities for self-financing:	NTCFAR	0.0	Indicates money available for investment. Formula: [Net operation activity/Investment outflow]
3	Cash flow sufficiency for current activities for settlement of obligations:	CFSFCA	-3.09366	Indicates money available for payment of debt and liabilities. Formula: [Net

				Operation/Activity outflow]
4	Profitability of positive cash flow ratio:	POCFR	0.024484	Indicates sufficient amount net Profit receive by CBN. Formula: [Net Investment/Cash inflow]

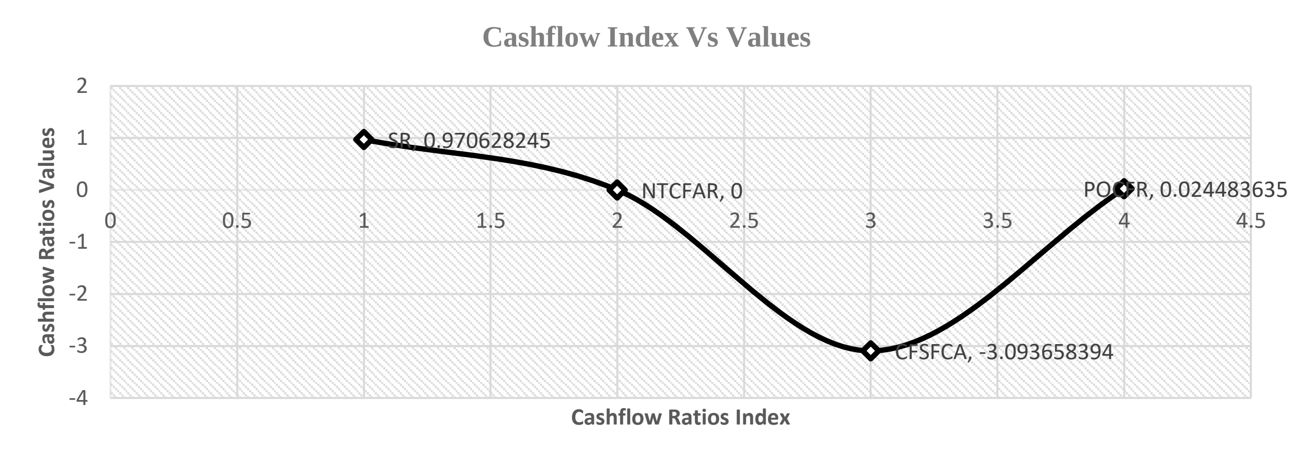

Figure 4.2: Graphical Representation of Cashflow ratios of State-2013

(iii) 2014 Net Cashflow Analysis
Table 4.5: 2014 Net Cashflow Analysis

DESCRIPTION	INFLOW	OUTFLOW	NET Cash flow
Net CBN Cash Inflow			46,642.44
Net CBN Operational Activities	41,238.34	54,329.14	-13,090.80
Net CBN Investment Activities	871.96	135.54	736.42
Net CBN Financial Activities	4,532.14	365.12	4,167.02
Net CBN Cashflow	46,642.44	54,829.80	-8,187.36
Net CBN Cash outflow			54829.8

Table 4.6: Results and Discussion

S/n	Cashflow Ratio Description	Cashflow Ratio Index	Ratio Values	Indication
1	The solvency ratio within the year under consideration:	SR	0.850676822	Indicates money available to take care of obligation. Formula: [Inflow/outflow]
2	Net cash flow adequacy ratio for	NTCFAR	-96.58255865	Indicates money available for investment. Formula: [Net operation

	current activities for self-financing:			activity/Investment outflow]
3	Cash flow sufficiency for current activities for settlement of obligations:	CFSFCA	-35.85341805	Indicates money available for payment of debt and liabilities. Formula: [Net Operation/Activity outflow]
4	Profitability of positive cash flow ratio:	POCFR	0.015788625	Indicates sufficient amount net Profit receive by CBN. Formula: [Net Investment/Cash inflow]

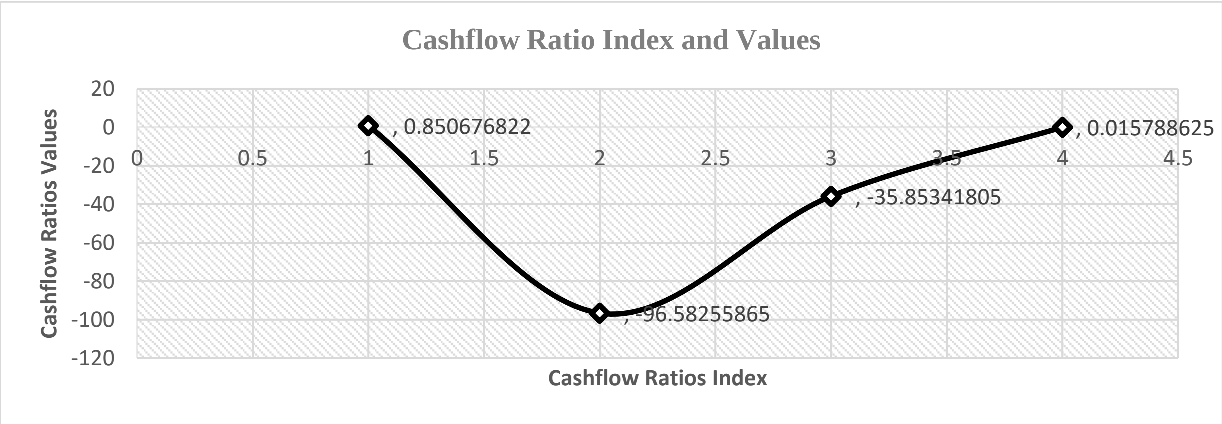

Figure 4.3: Graphical Representation of Cashflow ratios of State-2014

(iv) 2015 Net Cashflows Analysis

Table 4.7: 2015 Net Cashflows Analysis

DESCRIPTION	INFLOW	OUTFLOW	NET Cash flow
Net CBN Cash Inflow			33,529.47
Net CBN Operational Activities	26,939.69	33,032.40	-6092.71
Net CBN Investment Activities	697.04	0.00	697.04
Net CBN Financial Activities	5,892.74	5,319.57	573.17
Net CBN Cashflow	33,529.47	38,351.97	-4,822.50
Net CBN Cash outflow			38,351.97

Table 4.8: Results and Discussion

S/n	Cashflow Ratio Description	Cashflows Ratio Index	Ratio Values	Indication
1	The solvency ratio within the year under consideration:	SR	0.874256785	Indicates money available to take care of obligation. Formula: [Inflow/outflow]
2	Net cash flow adequacy ratio for current activities for self-financing:	NTCFAR	0.0	Indicates money available for investment. Formula: [Net operation activity/Investment outflow]
3	Cash flow sufficiency for current activities for settlement of obligations:	CFSFCA	-1.145338815	Indicates money available for payment of debt and liabilities. Formula: [Net Operation/Activity outflow]
4	Profitability of positive cash flow ratio:	POCFR	0.020788876	Indicates sufficient amount net Profit receive by CBN. Formula: [Net Investment/Cash inflow]

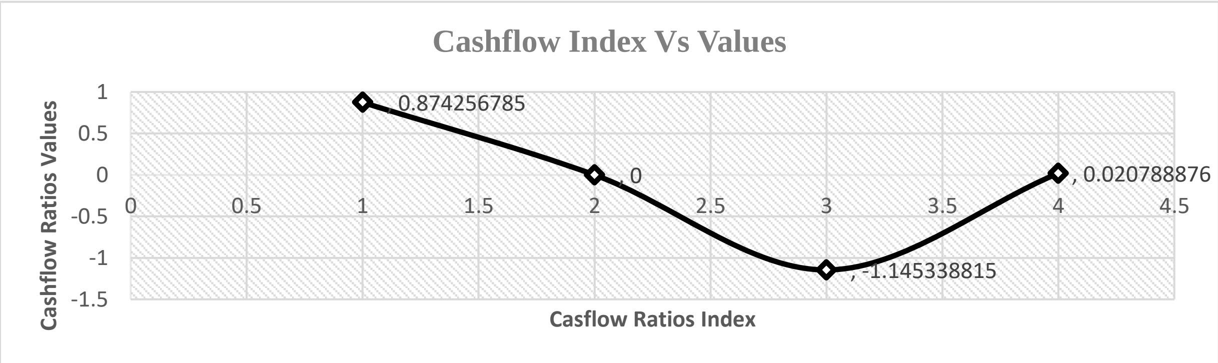

Figure 4.4: Graphical Representation of Cashflows ratios of State-2015

(v) 2016 Net Cashflow Analysis

Table 4.9: 2016 Net Cashflow Analysis

DESCRIPTION	INFLOW	OUTFLOW	NET Cash flow
Net CBN Cash Inflow			21,066.19
Net CBN Operational Activities	16,194.32	17,272.62	-1,078.30
Net CBN Investment Activities	574.75	141.71	433.04

Net CBN Financial Activities	4,297.12	5,749.83	-1,452.71
Net CBN Cashflow	21,066.19	23,164.16	-2,097.97
Net CBN Cash outflow			23,164.16

Table 4.10: Results and Discussion

S/n	Cashflow Ratio Description	Cashflow Ratio Index	Ratio Values	Indication
1	The solvency ratio within the year under consideration:	SR	0.909430344	Indicates money available to take care of obligation. Formula: [Inflow/outflow]
2	Net cash flow adequacy ratio for current activities for self-financing:	NTCFAR	-7.609201891	Indicates money available for investment. Formula: [Net operation activity/Investment outflow]
3	Cash flow sufficiency for current activities for settlement of obligations:	CFSFCA	-0.187535979	Indicates money available for payment of debt and liabilities. Formula: [Net Operation/Activity outflow]
4	Profitability of positive cash flow ratio:	PORCFR	0.020556161	Indicates sufficient amount net Profit receive by CBN. Formula: [Net Investment/Cash inflow]

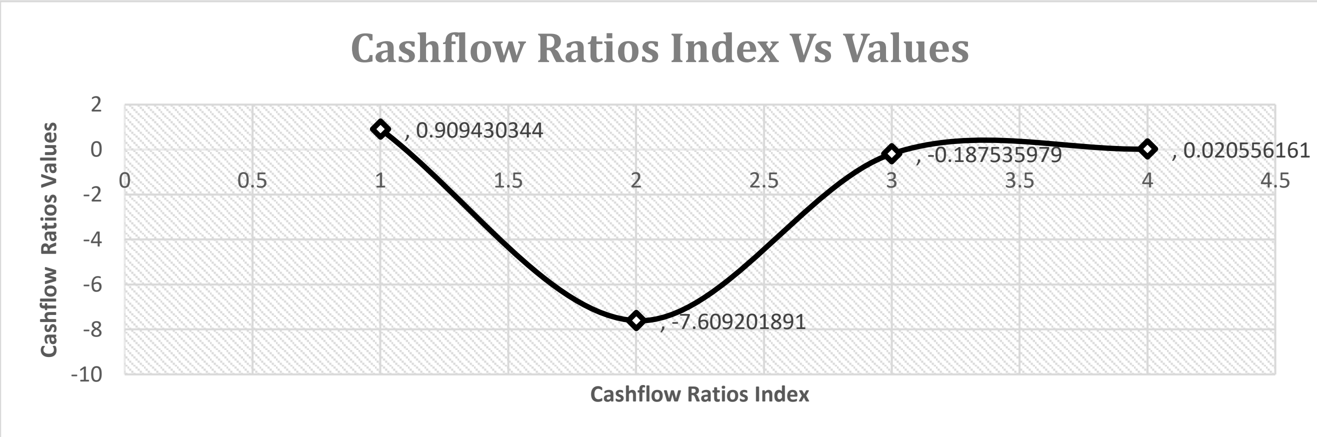

Figure 4.5: Graphical Representation of Cashflow ratios of State-2016

(vi) 2017 Net Cashflow Analysis

Table 4.11: 2017 Net Cashflow Analysis

DESCRIPTION	INFLOW	OUTFLOW	NET Cash flow
Net CBN Cash Inflow			42,172.16
Net CBN Operational Activities	19,270.63	28,759.33	-9,488.70
Net CBN Investment Activities	6,484.64	259.81	6224.83
Net CBN Financial Activities	16,416.89	1,533.64	14883.25
Net CBN Cashflow	42,172.16	30,552.78	11,619.38
Net CBN Cash outflow			30552.78

Table 4.12: Results and Discussion

S/n	Cashflow Ratio Description	Cashflow Ratio Index	Ratio Values	Indication
1	Solvency Ratio	SR	1.380305	Indicates money available to take care of obligation. Formula: [Inflow/outflow]
2	Net cash flow adequacy Ratio	NTCFAR	-36.5217	Indicates money available for investment. Formula: [Net operation activity/Investment outflow]
3	Cash flow sufficiency Ratio	CFSFCA	-6.18705	Indicates money available for payment of debt and liabilities. Formula: [Net Operation/Activity outflow]
4	Profitability of cash flow Ratio:	POCFR	0.147605	Indicates sufficient amount net Profit receive by CBN. Formula: [Net Investment/Cash inflow]

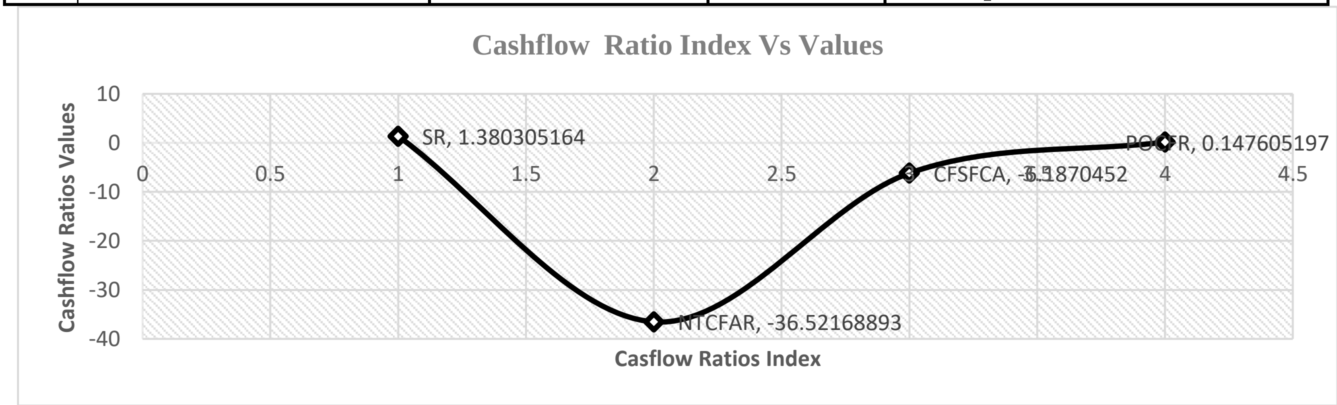

Figure 4.6: Graphical Representation of Cashflows ratios of State-2017

4.11 State's(2012, 2013, 2014, 2015, 2016 &2017) Health Status of CBN Cashflows

The future projection of cash flow indicates positives ratios (Healthy Cashflow) that will take care of obligation, investment liabilities and even better profits. The table below shows the State's health status of CBN Cashflows.

Table 4.13: State's health status of CBN Cashflows.

States (Assumed Current events)	Number of Cashflows Ratios above board	Cash inflow and cash outflow	Cashflow Ratios in terms of SR	Cashflow Healthy status as a function of columns-2, 3 and 4.
2012	4 out of 4	Inflow > outflow	SR >1	100
2013	3 out of 4	Inflow < outflow	SR <1	75
2014	2 out of 4	Inflow < outflow	SR <1	50
2015	3 out of 4	Inflow < outflow	SR <1	75
2016	2 out of 4	Inflow < outflow	SR <1	50
2017	2 out of 4	Inflow > outflow	SR >1	50

The percentage results indicate the level of cashflow operations, investment and financial activities performed by the CBN. Ultimately, healthy cash flow implies that cash flow ratios of solvency, adequacy, sufficiency and profitability must be greater than 1 and should not be allowed to slide below zero mark. The foregoing also indicates equilibrium in the operations, investment and financial activities of CBN Cashflows.

4.2 Markov Chain Data Analysis

The equations having satisfied Markova homogeneous chain are analyzed by Markov steady state. There two methods for solving the infinite-stage problem. The first method calls for evaluating all possible stationary polices of the decision problem. This is equivalent to an exhaustive enumeration process and can be used only if the number of stationary policies is reasonably small. The second method, called policy iteration, is generally more effective because it determines the optimum policy iteratively. Conversely, the second method was adopted for this research work, using Microsoft Excel Power Matrix as used by (Ohaji, E 2019) in River basin optimization Processes.

4.21 Simulation-1, for States: 2012, 2013, 2014, & 2015

Table 4.14 Step 1: Cashflows observed data

From: Present	**To: Future**			
Index	SR	NTCFAR	CFSFCA	POCFR
SR	1.33	18649	13.1	0.031233
NTCFAR	0.97	0	0	0.024484
CFSFCA	0.85	0	0	0.015789
POCFR	0.87	0	0	0.020789

Table 4.15 Step 2:Converts the matrix of step 1 to probability

From: Present	To: Future			
Index	SR	NTCFAR	CFSFCA	POCFR
SR	7.13E-05	0.999225	0.000702	1.67E-06
NTCFAR	0.97538	0	0	0.02462
CFSFCA	0.981763	0	0	0.018237
POCFR	0.976662	0	0	0.023338

In this probability table the sum of each row (row 1 to 4) must be equal to 1, to satisfy the Markovian chain criteria.

From Present	To Future			
Index	SR	NTCFAR	CFSFCA	POCFR
SR	X11	X12	X13	X14
NTCFAR	X21	X22	X23	X24
CFSFCA	X31	X32	X33	X34
POCFR	X41	X42	X43	X44

From: Present	To: Future			
Index	SR	NTCFAR	CFSFCA	POCFR
SR	7.13E-05	0.999225	0.000702	1.67E-06
NTCFAR	0.97538	0	0	0.02462
CFSFCA	0.981763	0	0	0.018237
POCFR	0.976662	0	0	0.023338

Hence, let the vector quantity represented as: ∏1, ∏2, ∏3, ∏4.

Therefore, the objective function represented as

Recall: лP = л

Therefore, the objective function stated below:

$$\begin{pmatrix} \Pi_1 & \Pi_2 & \Pi_3 & \Pi_4 \end{pmatrix} \times \begin{pmatrix} 7.13E\text{-}05 & 0.999225 & 0.000702 & 1.67E\text{-}06 \\ 0.97538 & 0 & 0 & 0.02462 \\ 0.981763 & 0 & 0 & 0.018237 \\ 0.976662 & 0 & 0 & 0.023338 \end{pmatrix} = \begin{pmatrix} \Pi_1, \\ \Pi_2, \\ \Pi_3, \\ \Pi_4. \end{pmatrix}$$ Equation 4.2

From the matrix arrangement, four (4) equations were generated as follows:

$7.13E\text{-}05\Pi_1 + 0.97538\Pi_2 + 0.981763\Pi_3 + 0.976662\Pi_4 = \Pi_1$ Equation 4.3

$0.999225\Pi_1 + 0\Pi_2 + 0\Pi_3 + 0\Pi_4 = \Pi_2$ Equation 4.4

$0.000702\Pi_1 + 0\Pi_2 + 0\Pi_3 + 0\Pi_4 = \Pi_3$ Equation 4.5

$1.67E\text{-}06\Pi_1 + 0.02462\Pi_2 + 0.018237\Pi_3 + 0.023338\Pi_4 = \Pi_4$ Equation 4.6

On solving the four equations simultaneously, the Optimum Policy values were obtained as follows: Optimum Policy Values: $\Pi_1 = 0.493818$, $\Pi_2 = 0.493389$, $\Pi_3 = 0.000347$, $\Pi_4 = 0.012446$

Alternatively, solving the above equation by applying Microsoft Excel Power Matrix as used by (Ohaji E, 2019) in River basin optimization Processes. The matrix below evaluated using Microsoft Excel Power, to the power of 400 iteration

$$\begin{pmatrix} 7.13E\text{-}05 & 0.999225 & 0.000702 & 1.67E\text{-}06 \\ 0.97538 & 0 & 0 & 0.02462 \\ 0.981763 & 0 & 0 & 0.018237 \\ 0.976662 & 0 & 0 & 0.023338 \end{pmatrix}^{400}$$ Equation 4.7

Markovian Iteration to the power of 400 using Microsoft Excel Power Matrix.

$$\begin{pmatrix} 0.493817896 & 0.493389398 & 0.000346628 & 0.012445952 \\ 0.493773103 & 0.493435293 & 0.00034666 & 0.012444816 \\ 0.493772805 & 0.493435591 & 0.00034666 & 0.012444808 \\ 0.493773043 & 0.493435353 & 0.00034666 & 0.012444814 \end{pmatrix}$$ Equation 4.8

However, in the 1st simulation analysis, the sum of each row must be equal to 1 to satisfy the Markovian Chain criteria. From the matrix above, the optimum policy was observed where a stationary point was reached (all the row values were deemed irreducible), as follows:

$\prod 1 =$ 0.493818

$\prod 2 =$ 0.493389

$\prod 3 =$ 0.000347

$\prod 4 =$ 0.012446

Optimum Policy Values from Simulation-1:

Optimum Policy Values: $\prod_1 = 0.493818$, $\prod_2 = 0.493389$, $\prod_3 = 0.000347$, $\prod_4 = 0.012446$

4.22 Simulation-2, States: 2014, 2015, 2016, & 2017

Table 4.16 Step 1: Cashflows observed data

From: Present	**To: Future**			
Index	SR	NTCFAR	CFSFCA	POCFR
SR	0.85	0	0	0.015789
NTCFAR	0.87	0	0	0.020789
CFSFCA	0.90943	0	0	0.020556
POCFR	1.380305	0	0	0.147605

Table 4.17 Step 2:Converts the matrix of step 1 to probability

From: Present	**To: Future**			
index	SR	NTCFAR	CFSFCA	POCFR
SR	0.981763	0	0	0.018237
NTCFAR	0.976662	0	0	0.023338

CFSFCA	0.977896	0	0	0.022104
POCFR	0.903394	0	0	0.096606

In this probability table, each row (rows 1 to 4) must sum to 1 to meet Markovian chain criteria. The total sum of probabilities in each row is 1. Converting table values into equations and solving them through simultaneous equations or using Microsoft Excel Power Matrix, as demonstrated by Ohaji E. in River Basin Optimization Processes (2019). Let the scalar quantity of the arrangement be:

From: Present To: Future

Index	SR	NTCFAR	CFSFCA	POCFR
SR	0.981763	0	0	0.018237
NTCFAR	0.976662	0	0	0.023338
CFSFCA	0.977896	0	0	0.022104
POCFR	0.903394	0	0	0.096606

$$P = \begin{pmatrix} 0.981763 & 0 & 0 & 0.018237 \\ 0.976662 & 0 & 0 & 0.023338 \\ 0.977896 & 0 & 0 & 0.022104 \\ 0.903394 & 0 & 0 & 0.096606 \end{pmatrix} \quad \text{Equation 4.9}$$

Hence, let the vector quantity represented as: ∏1, ∏2, ∏3, ∏4.

Therefore, the objective function represented as

Recall: лP = л

Therefore, the objective function stated below:

$$\begin{pmatrix} \prod 1 \prod 2 \prod 3 \prod 4 \\ \\ \\ \end{pmatrix} \times \begin{pmatrix} 0.981763 & 0 & 0 & 0.018237 \\ 0.976662 & 0 & 0 & 0.023338 \\ 0.977896 & 0 & 0 & 0.022104 \\ 0.903394 & 0 & 0 & 0.096606 \end{pmatrix} = \begin{pmatrix} \prod 1, \\ \prod 2, \\ \prod 3, \\ \prod 4. \end{pmatrix}$$ Equation 4.10

From the matrix arrangement four (4) equations were generated as follows:

$0.981763\prod_1 + 0.023338\prod_2 + 0.977896\prod_3 + 0.903394\prod_4 = \prod_1$ Equation 4.11

$0\prod_1 + 0\prod_2 + 0\prod_3 + 0\prod_4 = \prod_2$ Equation 4.12

$0\prod_1 + 0\prod_2 + 0\prod_3 + 0\prod_4 = \prod_3$ Equation 4.13

$0.018237\prod_1 0.023338\prod_2 + 0.022104\prod_3 + 0.096606\prod_4 = \prod_4$ Equation 4.14

On solving the four equations simultaneously, the Optimum Policy values were obtained as follows:

$\prod_1 = 0.980214059$, $\prod_2 = 0.0$, $\prod_3 = 0.0$, $\prod_4 = 0.01978668$

Alternatively, solving the above equation by applying Microsoft Excel Power Matrix as

The matrix below evaluated using Microsoft Excel Power, to the power of 400 iteration

$$\begin{pmatrix} 0.981763 & 0 & 0 & 0.018237 \\ 0.976662 & 0 & 0 & 0.023338 \\ 0.977896 & 0 & 0 & 0.022104 \\ 0.903394 & 0 & 0 & 0.096606 \end{pmatrix}^{400}$$ Equation 4.15

Markovian Iteration to the power of 400 using Microsoft Excel Power Matrix, gave the following irreducible solutions:

$$\begin{pmatrix} 0.980214059 & 0 & 0 & 0.01978668 \\ 0.980214059 & 0 & 0 & 0.01978668 \\ 0.980214 & 0 & 0 & 0.019786678 \\ 0.980214059 & 0 & 0 & 0.01978668 \end{pmatrix}$$ Equation 4.16

Therefore, the Microsoft Excel Power Matrix gave the same value Therefore, the optimum cash flow ratios Solutions or **Policy from simulation-2** are as follows:

Optimum Policy Values from Simulation-2:

Optimum Policy Solution: $\prod_1 = 0.980214059$, $\prod_2 = 0.0$, $\prod_3 = 0.0$, $\prod_4 = 0.01978668$

4.3 Model Optimization

This subsection addresses the mathematical optimization of the Markovian chain process, which involves obtaining the mean values from simulation-1 and simulation-2, leading to simulation-3. These mean values are then considered the optimized values for the Cashflow ratios policy or solution. Furthermore, the mean values of simulation-1 and simulation-2 define the Markovian Cashflow Ratio Monitoring Curve, as detailed below:

$\prod 1 =$ 0.737016
$\prod 2 =$ 0.246695
$\prod 3 =$ 0.000173
$\prod 4 =$ 0.016116

4.31 Markovian Cashflow Ratio Monitoring Curve (MCFRMC)

This subsection developed Markovian Cashflow Ratio Monitoring Curve (MCFRMC) as follows:

Table 4.18 Optimum Policy Values

Index	Optimum Policy	Cashflow Ratios Values
SR	$\prod 1 =$	0.737016
NTCFAR	$\prod 2 =$	0.246695
CFSFCA	$\prod 3 =$	0.000173
PORCFR	$\prod 4 =$	0.016116

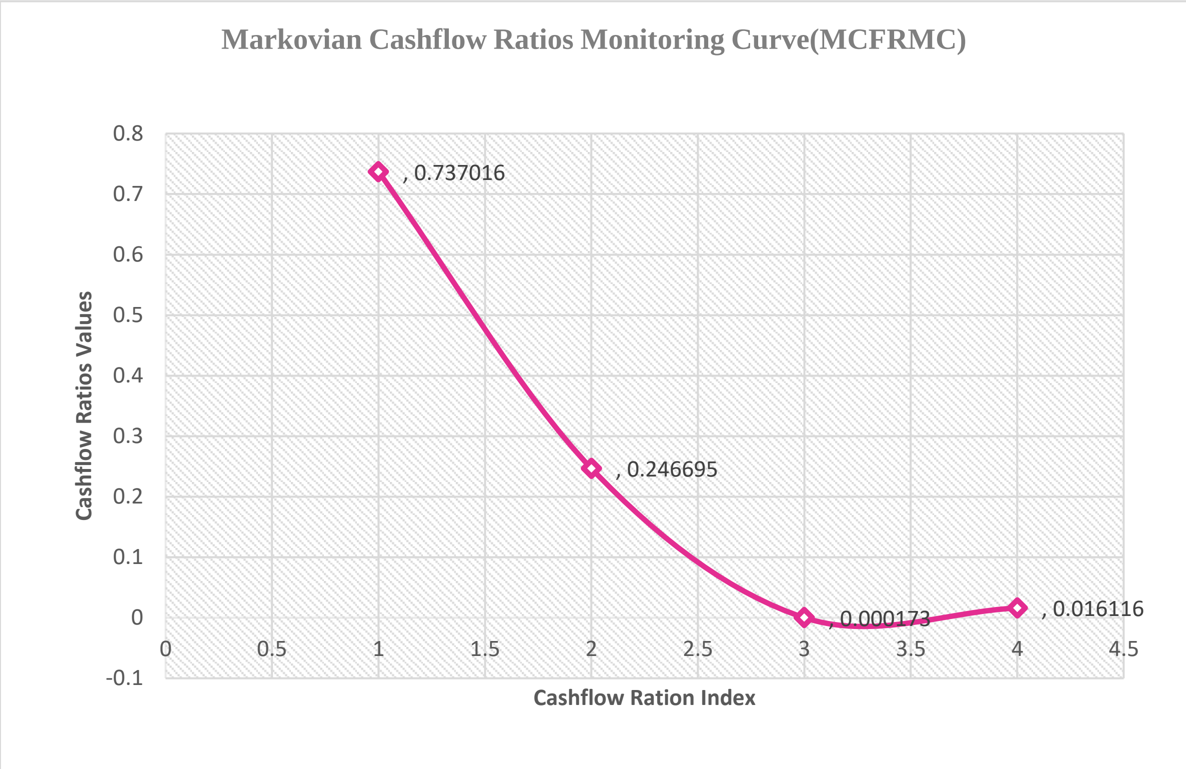

Figure 4.7: Markovian Cashflow Ratios Monitoring Curve (MCFRMC)

Simulation averages determine the predictive optimal policy values, forming the Markovian Cashflow Ratio Monitoring Curve (MCFRMC). CBN statisticians will use this curve for future cash flow ratio monitoring. For demonstration, it will compare cash flow ratios between State-2012-2017, illustrating its present and future applicability.

4.32 Model Validation

(i) SIMULATION-1

Substitute the policy solution of simulation-1 into equation 4.3

Where:

$\prod_1 = 0.493818$, $\prod_2 = 0.493389$, $\prod_3 = 0.000347$, $\prod_4 = 0.012446$

$7.13E\text{-}05\prod_1 + \; 0.97538\prod_2 + 0.981763\prod_3 + 0.976662\prod_4 = \prod_1$ Equation 4.3

(0.0000713*0.493818) +(0.97538*0.493389) +(0.981763*0.000347) +(0.97662*0.012446) = 0.493773

The Validation of equation 4.3 of simulation-1 confirmed that $\prod_1 = 0.493773$

(ii) SIMULATION-2

Substitute the policy solution of simulation-2 into equation 4.11

Where:

$\prod_1 = 0.980214059$, $\prod_2 = 0.0$, $\prod_3 = 0.0$, $\prod_4 = 0.01978668$

$0.981763\prod_1 + 0.023338\prod_2 + 0.977896\prod_3 + 0.903394\prod_4 = \prod_1$ Equation 4.11

(0.981763*0.980214059) +(0.023338*0) +(0.977896*0) +(0.903394*0.01978668) = 0.980213
The Validation of equation 4.11 of simulation-2 confirmed that $\prod_1 = 0.980213$
The simulation processes 1 and 2 conformed to the Markovian Model which stated that:
$\prod_1 + \prod_2 + \prod_3 + \prod_4 = 1$, this conformed to the 3rd constrain of Markovian Chain Model.

Table 4.19 Simulation-1

Policy	Model	Prototype
∏1	0.493818	9.75E-01
∏2	0.493389	0.999225
∏3	0.000347	0.000702
∏4	0.012446	1.67E-06

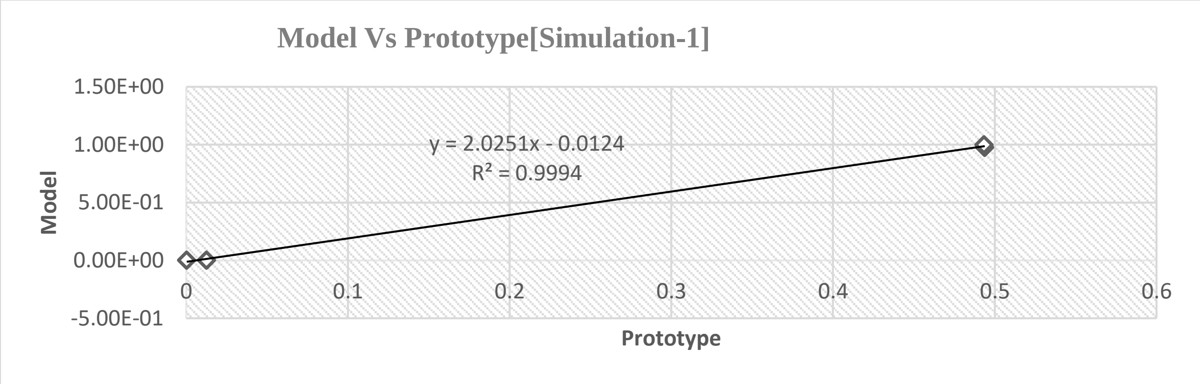

Figure 4.8: Validation of the Model [Simulation-1] with the Prototype

Table 4.20 Simulation-2

Policy	Model	Prototype
∏1	0.98021306	0.981763
∏2	0	0
∏3	0	0
∏4	0.019787676	0.018237

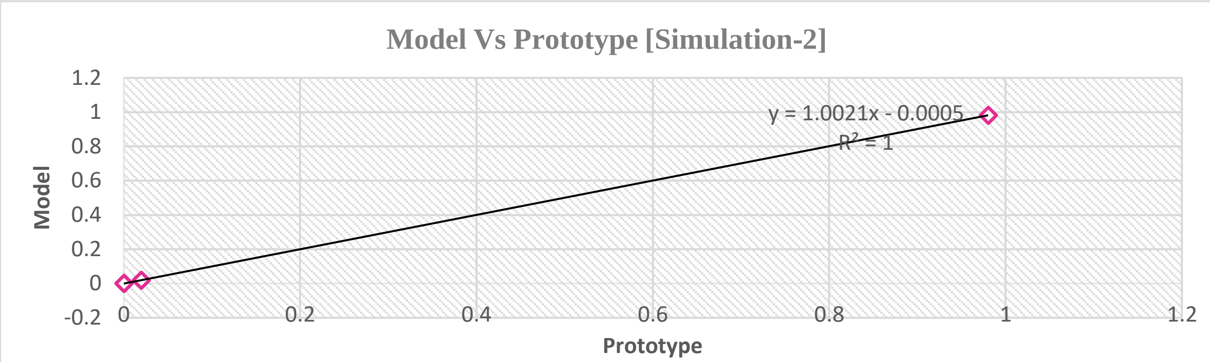

Figure 4.9: Validation of the Model [Simulation-2] with the Prototype

4.4 Application of Cashflow Ratio MCFRMC

In this subsection cashflow ratios of States- 2012 to 2017 were evaluated Using MCFRMC. Comparing cashflow ratio as observed in the various States with Final Simulation (Future-Probability prediction Model)

Table 4.21: Comparing State-2012[Prototype] and MCFRMC [Model]

Index	SR	NTCFAR	CFSFCA	POCFR
MCFMC(Future)	0.737016	0.246695	0.000173	0.016116
State-2012	1.327055	18648.79	13.10381	0.03123

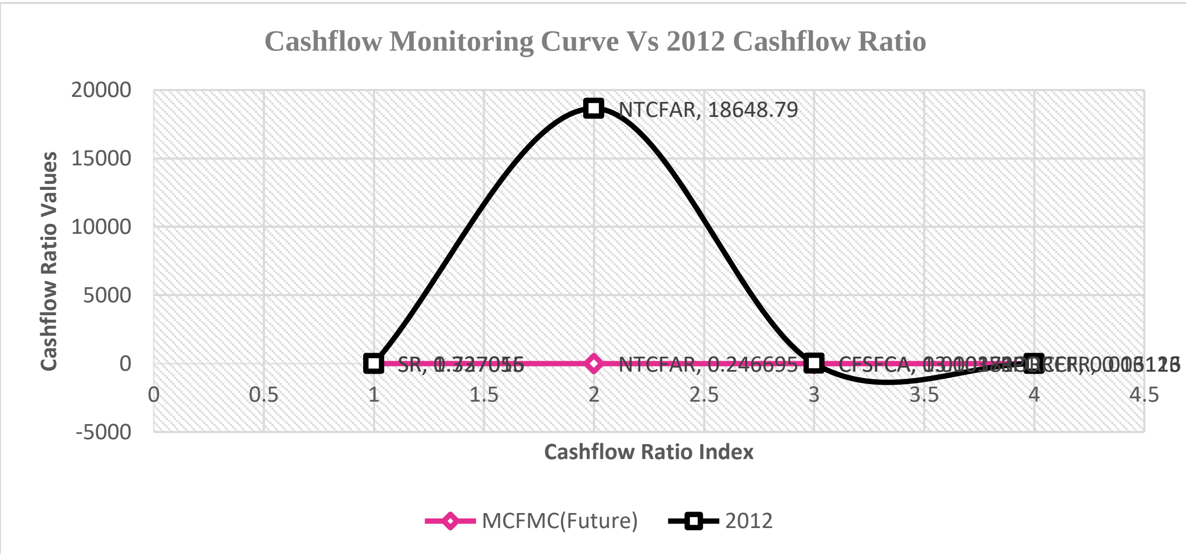

Figure 4.10: Comparing MCFRMC [Model] with the State-2012[Prototype]

The black curve represents cash flow ratios in State-2013, while the optimal policy curve is the red line, termed the Markovian Cash Flow Monitoring Curve (MCFMC). This curve sets a minimum threshold for cash flow ratios, denoted by the red line, allowing ratios to go above but not below it. The red curve values are: SR = 0.737016, NTCFAR = 0.246695, CFSFCA = 0.000173, POCFR = 0.016116. In contrast, the black curve values are: SR = 1.327055, NTCFAR = 18648.79, CFSFCA = 13.10381, POCFR = 0.03123.

Table 4.22: Comparing State: 2013[Prototype] and MCFRMC [Model]

Index	SR	NTCFAR	CFSFCA	POCFR
MCFMC(Future)	0.737016	0.246695	0.000173	0.016116
State-2013	0.970628	0	-3.09366	0.024484

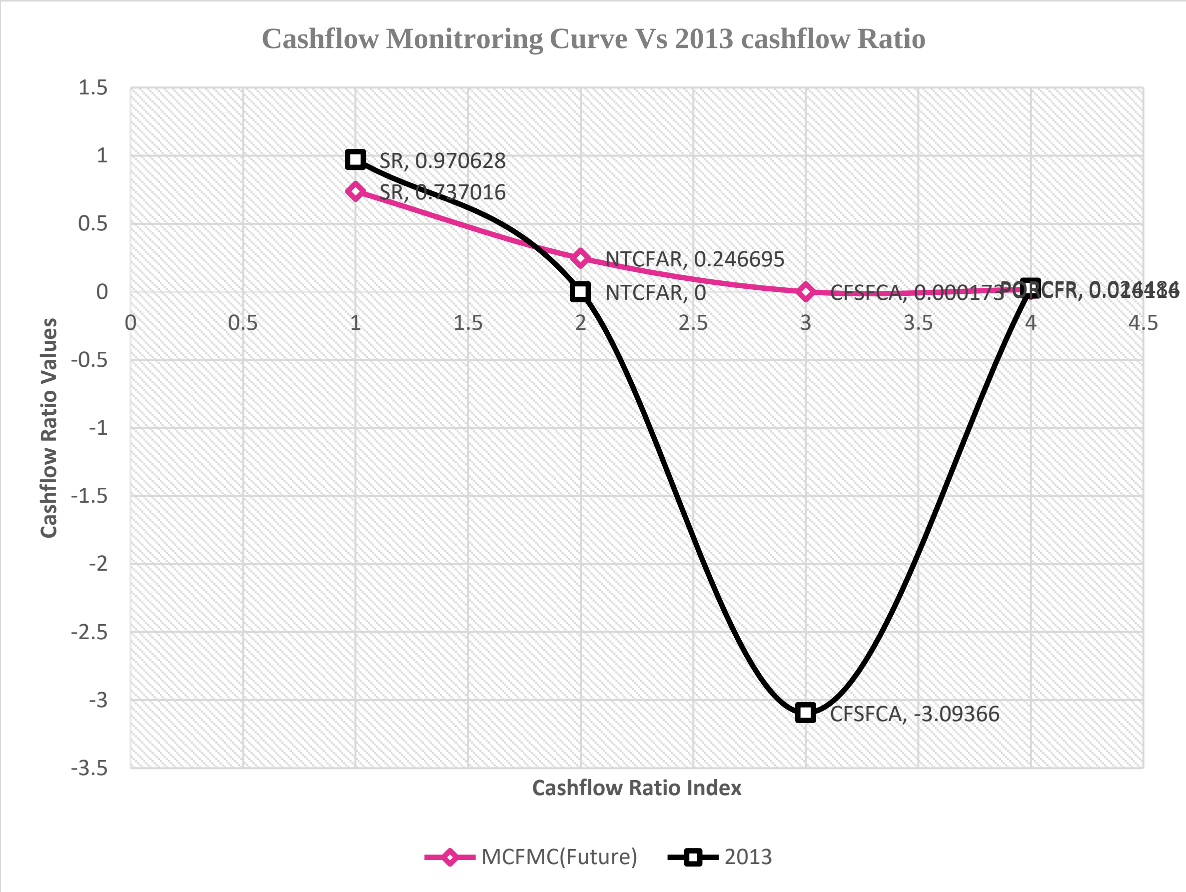

Figure 4.11: Comparing MCFRMC [Model] with the State-2013[Prototype]

The black curve depicts cash flow ratios in State-2013, and the optimal policy curve is the red line, known as the Markovian Cash Flow Monitoring Curve (MCFMC). This curve sets a minimum threshold for cash flow ratios, preventing them from falling below the values indicated by the red line. However, ratios are allowed to exceed the red curve values. The red curve values are SR = 0.737016, NTCFAR = 0.246695, CFSFCA = 0.000173, POCFR = 0.016116. The black curve values are SR = 0.970628, NTCFAR = 0.0, CFSFCA = -3.09366, POCFR = 0.024484.

Table 4.23: Comparing State: 2014[Prototype] and MCFRMC [Model]

Index	SR	NTCFAR	CFSFCA	POCFR
MCFMC(Future)	0.737016	0.246695	0.000173	0.016116
State-2014	0.850676822	-96.58255865	-35.8534	0.015789

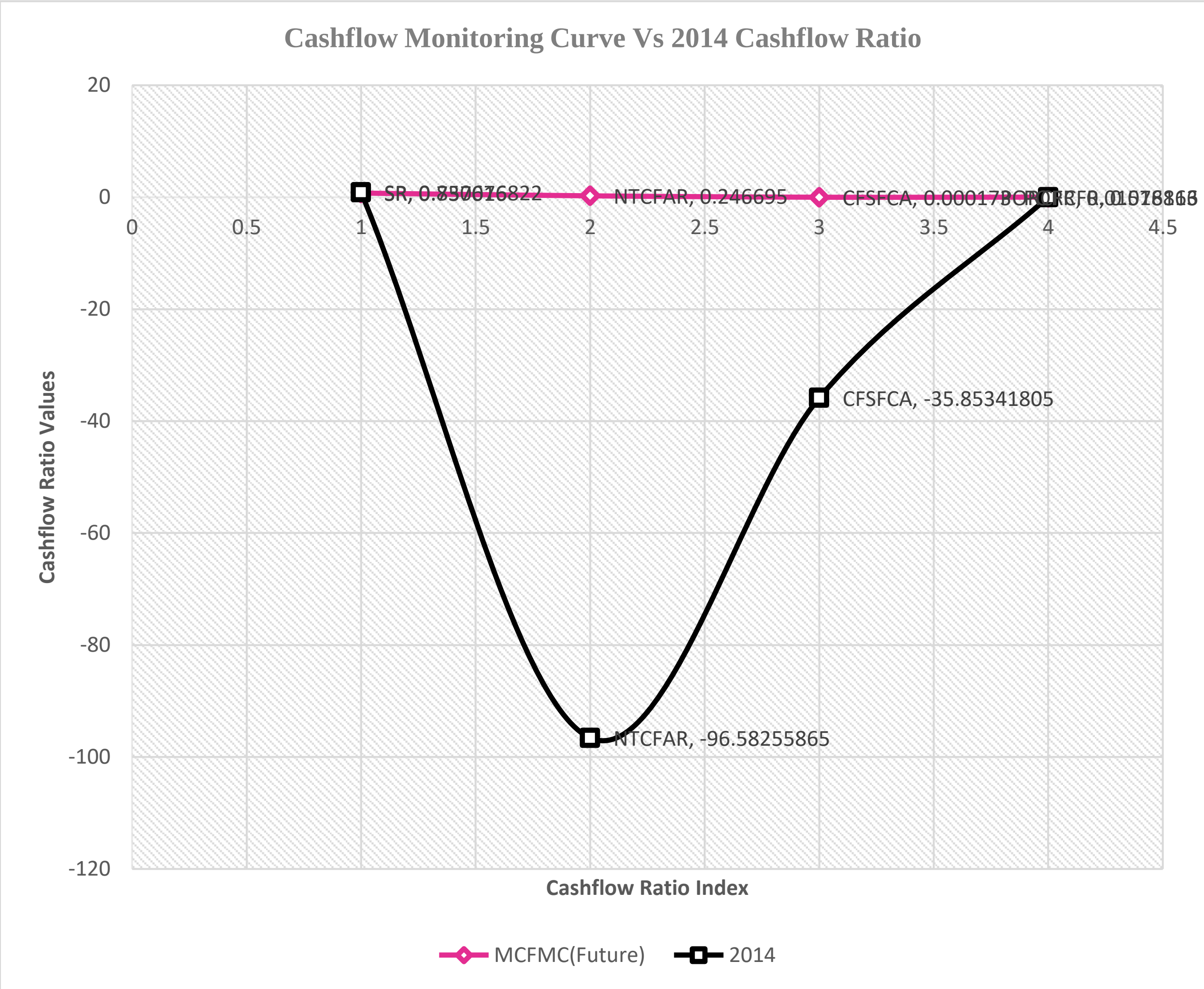

Figure 4.12: Comparing MCFRMC [Model] with the State-2014[Prototype]

The black curve reflects cash flow ratios in State-2014, and the optimal policy curve is the red line, termed the Markovian Cash Flow Monitoring Curve (MCFMC). This curve sets a minimum threshold for cash flow ratios, preventing them from falling below the values indicated by the red line. However, ratios are allowed to exceed the red curve values. The red curve values are SR = 0.737016, NTCFAR = 0.246695, CFSFCA = 0.000173, POCFR = 0.016116. The black curve values are SR = 0.850676822, NTCFAR = -96.58255865, CFSFCA = -35.8534, POCFR = 0.015789.

Table 4.24: Comparing State: 2015[Prototype] and MCFRMC [Model]

	SR	NTCFAR	CFSFCA	POCFR
MCFMC(Future)	0.737016	0.246695	0.000173	0.016116
State-2015	0.874256785	0	-1.14534	0.020789

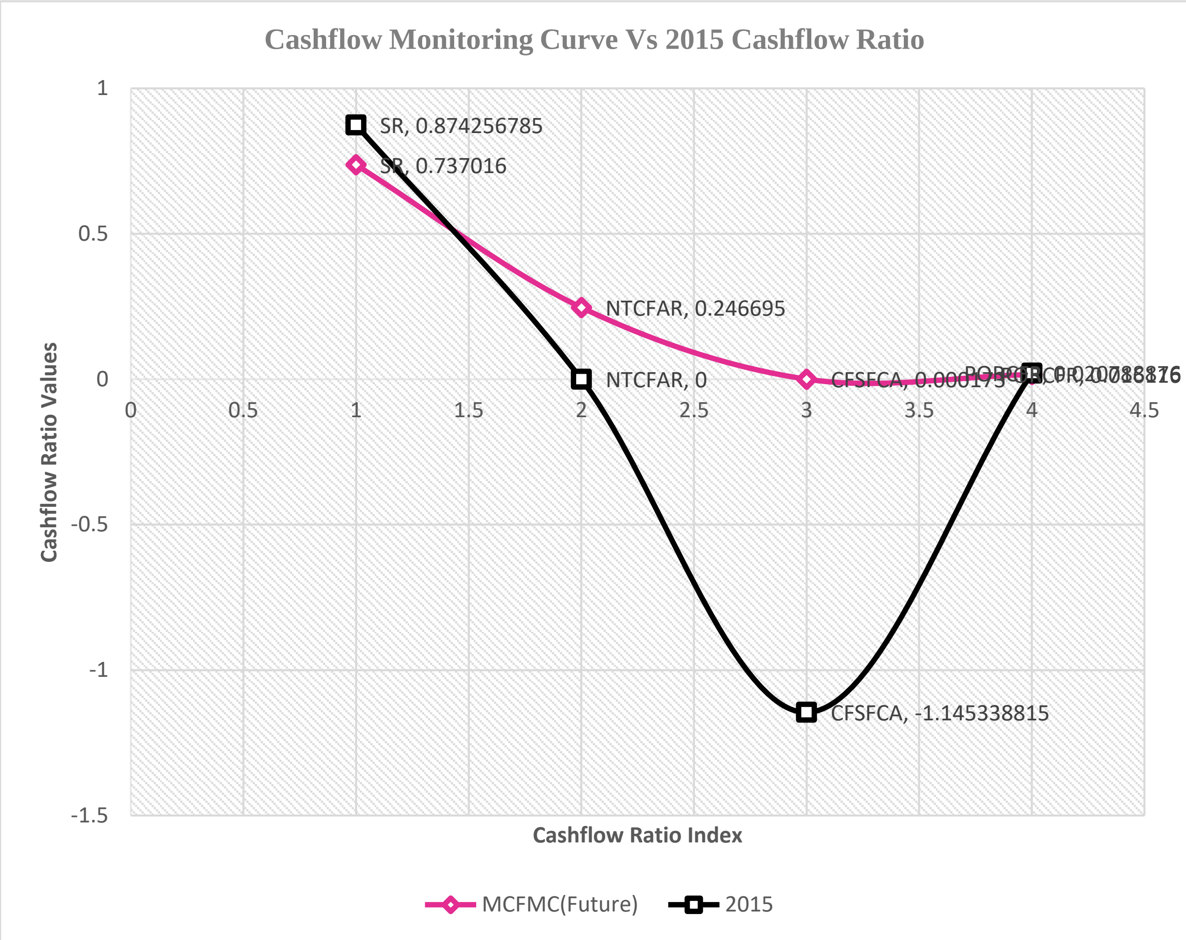

Figure 4.13: Comparing MCFRMC [Model] with the State-2015[Prototype]

The black curve reflects cash flow ratios in State-2015, with the optimal policy shown by the red line, termed the Markovian Cash Flow Monitoring Curve (MCFMC). This curve establishes a minimum threshold for cash flow ratios, allowing them to exceed but not fall below the values indicated by the red line. The red curve values are SR = 0.737016, NTCFAR = 0.246695, CFSFCA = 0.000173, POCFR = 0.016116. The black curve values are SR = 0.874256785, NTCFAR = 0.0, CFSFCA = -1.14534, POCFR = 0.020789.

Table 4.25: Comparing State: 2016[Prototype] and MCFRMC [Model]

Index	SR	NTCFAR	CFSFCA	POCFR
MCFMC(Future)	0.737016	0.246695	0.000173	0.016116
State-2016	0.909430344	-7.609201891	-0.187536	0.020556

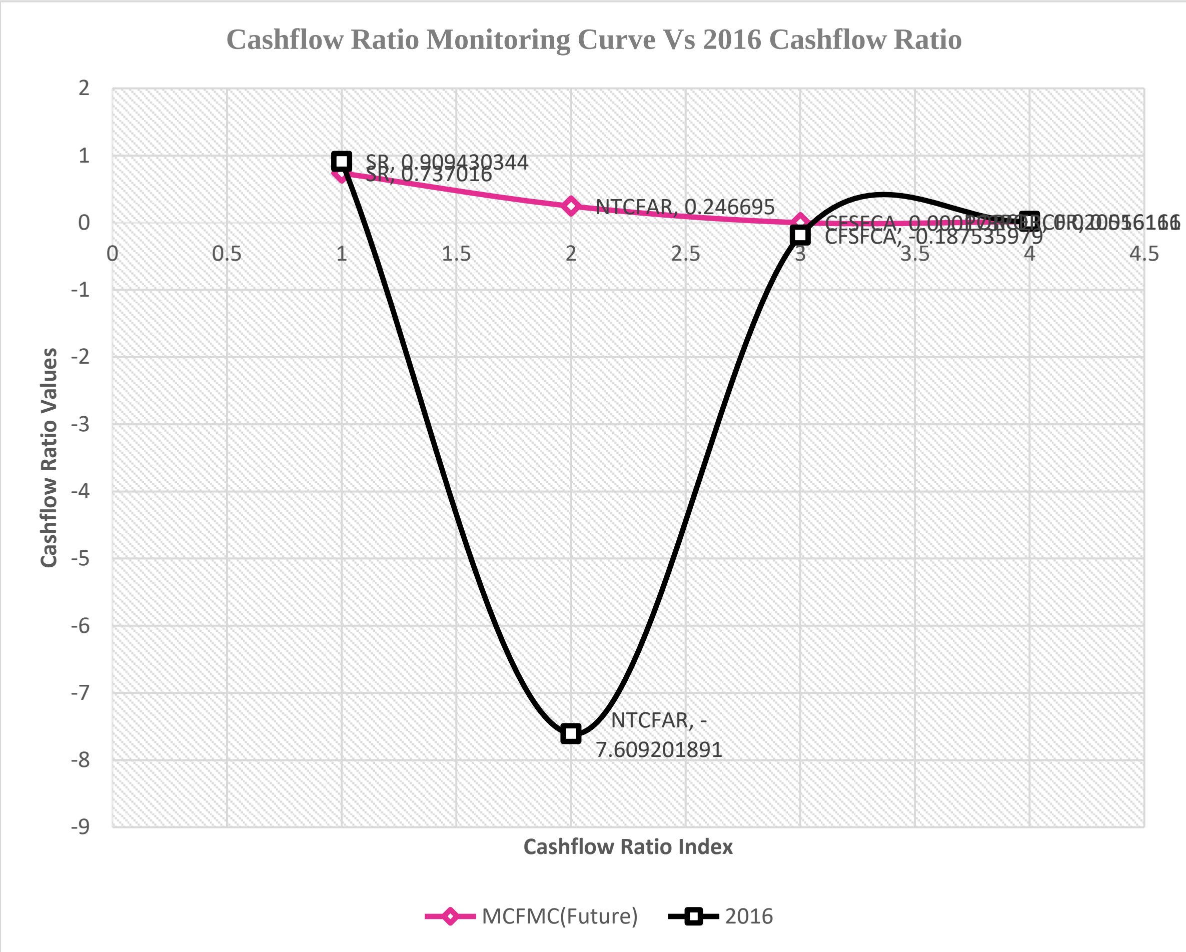

Figure 4.14: Comparing MCFRMC [Model] with the State-2016[Prototype]

The black curve illustrates cash flow ratios in State-2016, and the optimal policy curve is the red line, denoted as the Markovian Cash Flow Monitoring Curve (MCFMC). This curve establishes a minimum threshold for cash flow ratios, allowing them to exceed but not fall below the values indicated by the red line. The red curve values are SR = 0.737016, NTCFAR = 0.246695, CFSFCA = 0.000173, POCFR = 0.016116. The black curve values are SR = 0.909430344, NTCFAR = -7.609201891, CFSFCA = -0.187536, POCFR = 0.020556.

Table 4.26: Comparing State-2017[Prototype] and MCFRMC [Model]

Index	**SR**	**NTCFAR**	**CFSFCA**	**POCFR**
MCFMC(Future)	0.737016	0.246695	0.000173	0.016116
State-2017	1.380305	-36.5217	-6.18705	0.147605

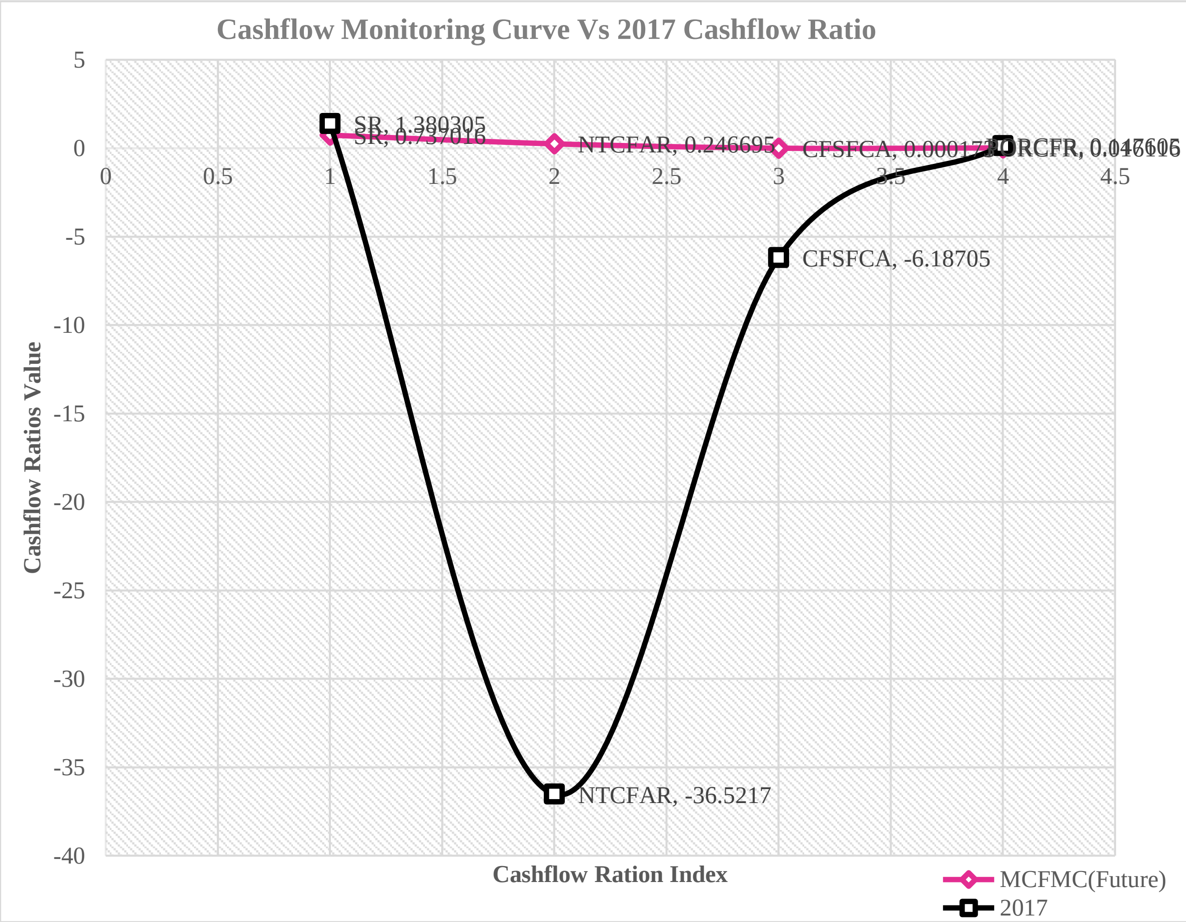

Figure 4.15: Comparing MCFRMC [Model] with the State-2017[Prototype]

The black curve illustrates cash flow ratios in State-2017, and the optimal policy curve is the red line, termed the Markovian Cash Flow Monitoring Curve (MCFMC). This curve sets a minimum threshold for cash flow ratios, allowing them to exceed but not fall below the values indicated by the red line. The red curve values are SR = 0.737016, NTCFAR = 0.246695, CFSFCA = 0.000173, POCFR = 0.016116. The black curve values are SR = 1.380305, NTCFAR = -36.5217, CFSFCA = -6.18705, POCFR = 0.147605.

4.5 Cashflows allocation

This subsection deals with modeling of cash inflow and outflow of operation, investment and financial activities. Similar modeling process of section 4.4 was applied in this section. Hence the model came up with a strategic policy of cash allocation to the foregoing activities.

4.5.1 Cash inflows allocation Modeling

Let's consider States, 2013 to 2015

Table 4.27: Prototype Cash inflow in States-2013 - 2015

DESCRIPTION	2013-INFLOW	2014-INFLOW	2015-INFLOW
Net CBN Cash Inflow			
Net CBN Operational Activities	40064.74	41,238.34	26,939.69
Net CBN Investment Activities	1005.55	871.96	697.04
Net CBN Financial Activities	0	4,532.14	5,892.74

Table 4.28: Probability of Prototype Cash inflow in States-2013 - 2015

DESCRIPTION	2013-INFLOW	2014-INFLOW	2015-INFLOW
Net CBN Cash Inflow			
Net CBN Operational Activities	0.37	0.38	0.25
Net CBN Investment Activities	0.39	0.34	0.27
Net CBN Financial Activities	0.00	0.43	0.57

Table 4.29: Simulation output at 400 Iteration

DESCRIPTION	INFLOW	INFLOW	INFLOW
Net CBN Cash Inflow			
Net CBN Operational Activities	0.24	0.38	0.38
Net CBN Investment Activities	0.24	0.38	0.38
Net CBN Financial Activities	0.24	0.38	0.38

Considering State 2013, the allocation of cash inflow values were stated in the 5th column of the table 4.30

Table 4.30: Future Predicted Allocation in Percentage

	Future Allocation	Expressed in ratio	Expressed in %age	Let consider inflow = 41,070.29
CBN Operational Activities	0.24	0.24	23.75	9752.886
CBN Investment Activities	0.38	0.38	38.36	15754.66
CBN Financial Activities	0.38	0.38	37.89	15562.74
Total	1	1	100%	

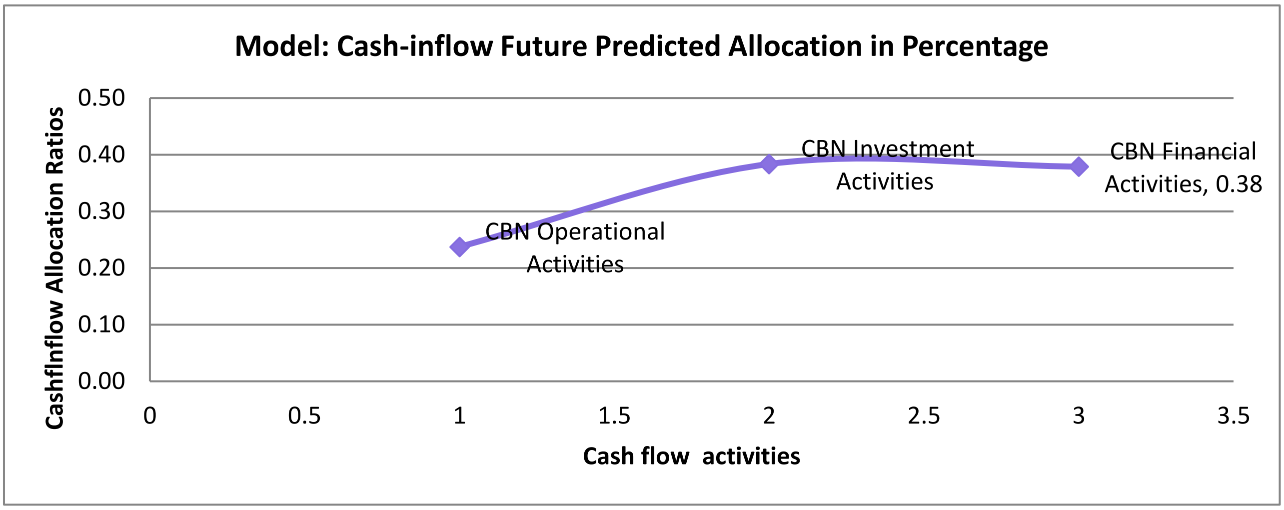

Figure 4.16: Graphical representation of the Model: Cash-inflow Future Predicted Allocation in Percentage

4.5.1.1 Discussion and Results

For a given cash inflow, cash allocation involves determining ratios or percentages, a process known as policy iteration. The modeled cash inflow allocation strategy is:
(i) Operations receive 24%.. (ii) Investments receive 38%., (iii) Financial activities receive 38%.

4.5.2 Cash outflows allocation Modeling

Similarly, let's consider states, 2013 to 2015

Table 4.31: Prototype Cash outflow in States-2013 - 2015

DESCRIPTION	OUTFLOW	OUTFLOW	OUTFLOW
Net CBN Cash Inflow			
Net CBN Operational Activities	41,763.87	54,329.14	33,032.40
Net CBN Investment Activities	0	135.54	0
Net CBN Financial Activities	549.23	365.12	5,319.57

Table 4.32: Probability of Prototype Cash outflow in States-2013 - 2015

DESCRIPTION	OUTFLOW	OUTFLOW	INFLOW	
Net CBN Cash Inflow				
Net CBN Operational Activities	0.32	0.42	0.26	
Net CBN Investment Activities	0.00	1.00	0.00	
Net CBN Financial Activities	0.09	0.06	0.85	

Table 4.33 Simulation output at 400 Iteration of Cash outflow

DESCRIPTION	OUTFLOW	OUTFLOW	OUTFLOW
Net CBN Cash Inflow			
Net CBN Operational Activities	0.00	1.00	0.00
Net CBN Investment Activities	0	1	0
Net CBN Financial Activities	0.00	1.00	0.00

Considering State 2013, the allocation of cash outflow values were stated in the 5th column of the table 4.34

Table 4.34: Model Cash outflow in States-2013 - 2015

	Future Allocation	**Expressed in ratio**	**Expressed in %age**	**Let consider outflow = 42,313.10**
CBN Operational Activities	0	0	0	0
CBN Investment Activities	1	1	100	42,313.10
CBN Financial Activities	0	0	0	0
Total	**1**	**1**		**42,31 3.10**

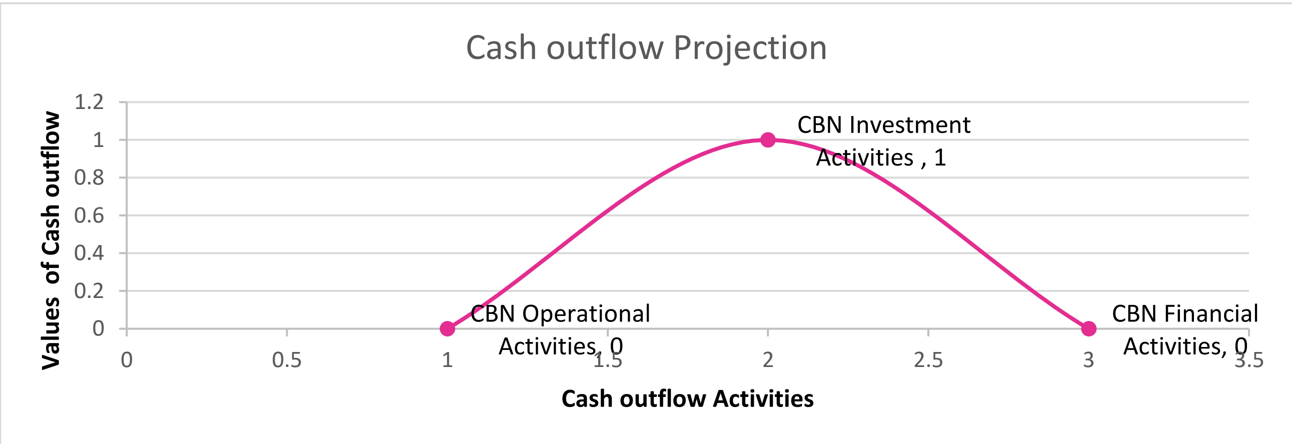

Figure 4.17; Graphical representation of Cash-outflow Projection

4.5.2.1 Discussion and Results

For a given cash outflow, cash allocation involves determining ratios or percentages, a process known as policy iteration. The modeled cash outflow allocation strategy is:
(i) Operations receive 0%.. (ii) Investments receive 1%., (iii) Financial activities receive 0%.

4.5.3 Optimal policy strategies of Cashflows Allocation
How do CBN operators allocate cash inflow to indicators such as operating, investment, and financial activities? Markovian decision theory provides a solution, as illustrated by the model output depicted in table 4.37 and figure 4.20.

4.5.4 Superimposing Cash-inflow projected and Cash-outflow Projected Allocations

Net CBN Cash Inflow & Outflow		
DESCRIPTION	**CASHINFLOW**	**CASHOUTFLOW**
Net CBN Operational Activities	0.24	0
Net CBN Investment Activities	0.38	1
Net CBN Financial Activities	0.38	0

Table: 4.35: Net CBN Cash Inflow & Outflow allocation Rations

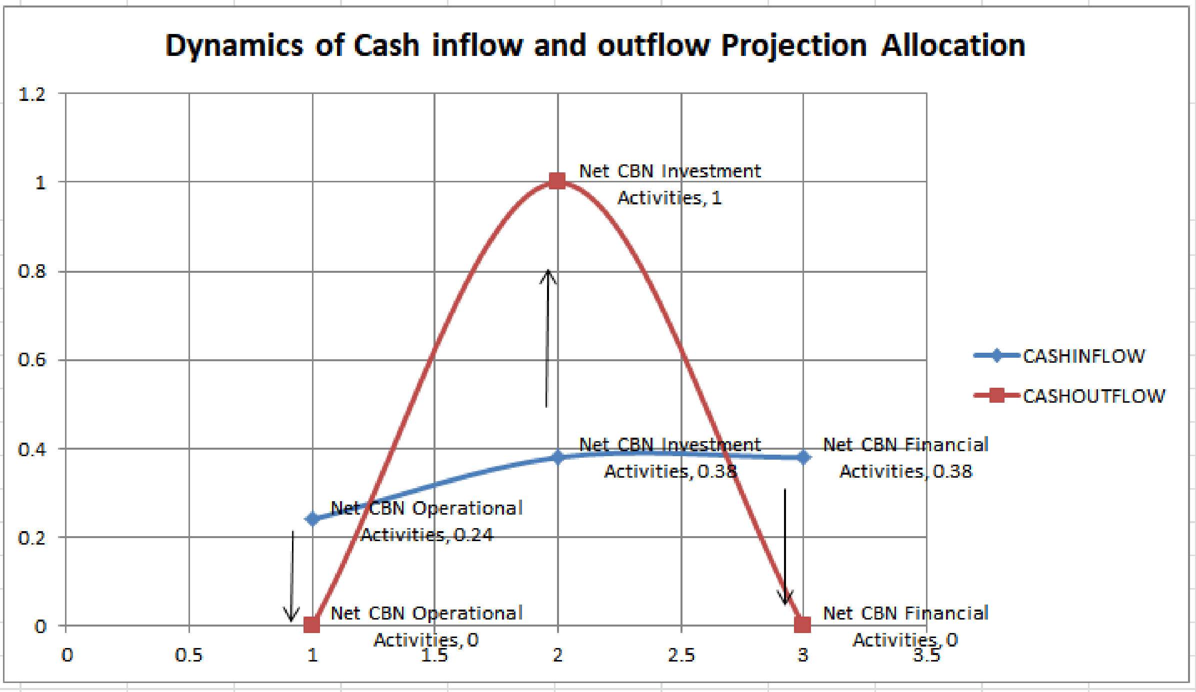

Figure 4.18: Dynamics of Cash Inflow and Outflow Projection Allocation

This model enhances cash-inflow allocation. From the graph:
(i) Cash allocated to operational and financial activities had minimal long-term monetary returns. (ii) Investment activity generated significant profits, approaching 100% performance.

Table 4.36: Model Validation of Cash inflow Allocation

Observed State-2013 [Prototype]	Projected State-2013 [Model]
41,238.34	9792.786
871.96	334.4856
4,532.14	1717.359

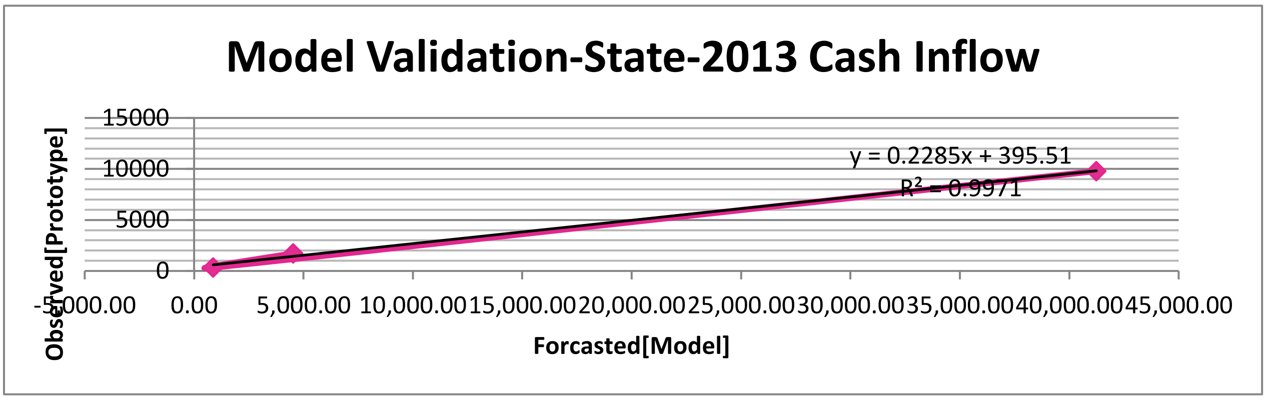

Figure 4.19: Model Validation State-2013 Cash Inflow

Table 4.37: Model Validation of Cash inflow Allocation

Observed State-2013 Outflow [Prototype]	**Projected State-2013 outflow [Model]**
41,238.34	2.47885E-17
871.96	0
4,532.14	2.06815E-18

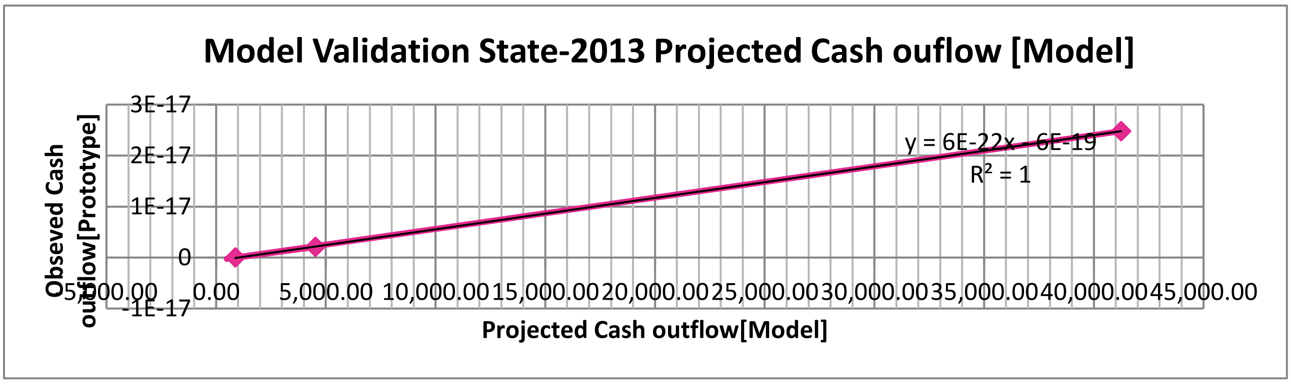

Figure 4.20: Model Validation State-2013 Cash outflow

4.6 Discussion

This research demonstrates the effectiveness of the Markov chain forecasting model in predicting future Cash Flow Ratios. The model, with its random walk in the transition matrix, produces more reliable results compared to similar models, aligning with prior studies by Piccardi et al., Hazra et al., and Tserenjigmid. The research highlights the critical role of developing a comprehensive yet simple predictive model for solving complex forecasting problems, offering valuable insights for improving cash flow ratios forecasting. The analysis results suggest that decision-makers find it easily understandable, requiring modest computation. For future forecasting, using Markovian chains is recommended for gaining better insights into cash flow behavior.

5.0 Conclusion

In this research paper, a successful Markov chain method has been developed to predict the future behavior of cashflow ratios and cashflow indexes, influenced entirely by stochastic factors. The study focuses on the CBN cashflow problem spanning from January 2012 to December 2017, involving six distinct stages. The cashflow performance in Stage-2012 was exceptional, reflecting 100% healthy cash flow ratios. In contrast, Stages-2013 and 2015 exhibited 75% healthy cashflow, while Stages-2014, 2016, and 2017 reflected 50% healthy cash flow. The behavior of cashflow ratios in Stage-2012 was evaluated using the Markov chain to forecast future ratios. The predicted results, expressed in terms of the probability of states in cashflow ratios and variable indexes, indicated forecasted cashflow ratios such as SR = 0.737016, NTCFAR = 0.246695, CFSFCA = 0.000173, and POCFR = 0.016116.

Validation of the prototype and model resulted in a coefficient equal to 1.0, indicating a 100% higher performance of the model compared to the prototype. Subsequent research unveiled a strategic cash-inflow policy allocation to cashflow indicators, allocating 24% to operational activities, 38% to investment activities, and 38% to financial activities. Meanwhile, the optimal cash-outflow strategy suggested that operational and financial activities tend towards 0%, while investment cash-outflows tend towards 100%.

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